1.   **NO CALCULATORS OR NOTES ALLOWED**
2.   **UNLESS STATED OTHERWISE, YOU MUST SIMPLIFY ALL ANSWERS**
3.   **SHOW PROPER CALCULUS LEVEL WORK TO JUSTIFY YOUR ANSWERS**

A certain economist believes that the rate at which a person's wealth changes is proportional to the difference   **SCORE:** 2 / 3 PTS
between their country's median wealth and their own wealth. Assuming that median wealth is a constant (W_{MEDIAN}), and that wealthy people
(people with a lot of wealth) tend to get wealthier, write a differential equation for the wealth $W(t)$ of a wealthy person at time t.
NOTE: The sign of all constants should be stated clearly.

$W(t) =$ wealth of a person at time t

$$\frac{dW}{dt} = k\,(W - W_{MEDIAN})$$

(2)

What does the Existence and Uniqueness Theorem tell you about possible solutions to the initial value problem   **SCORE:** 4 / 4 PTS
$\left(\frac{dy}{dx}\right)^3 + 1 = y, \quad y(3) = 1$? **Justify your answer properly, but briefly.**

$\left(\frac{dy}{dx}\right)^3 = y - 1 \qquad (3, 1)$

$\frac{dy}{dx} = (y-1)^{1/3}$ ①

$f(x, y) = (y-1)^{1/3}$

$\frac{df}{dy} = \frac{1}{3}(y-1)^{-2/3}$ ①

$= \frac{1}{3(y-1)^{2/3}}$

The function $\frac{df}{dy}$ is not defined at the point $(3, 1)$ ①
and therefore it is not continuous in an open region around
$(3, 1)$

E & U tells us nothing ①

Consider the IVP $y' = x(y-1), \quad y(2) = 6$. Use Euler's method with $h = 0.1$ to estimate $y(2.2)$.   **SCORE:** 4 / 4 PTS

$x_0 = 2 \quad y_0 = 6$

$Y_{n+1} = Y_n + f(x_n, Y_n)h$

$Y_1 = Y_0 + (x_0(Y_0 - 1))h = 6 + (2(5))(0.1)$ ①
$= 6 + 1 = 7$ ①

$Y_2 \approx Y(2 + 2 \times 0.1) \approx Y(2.2) = 7 + [(2.1)(6)](0.1)$ ①

$Y(2.2) = 7 + \frac{12.6}{10} = 7 + 1.26 = 8.26$

$Y(2.2) \approx 8.26$ ①

Consider the autonomous DE $y' = (2-y)^3(6-y)^2$.

[a] Find all equilibrium solutions of the DE and classify each as stable, unstable or semi-stable. **You must draw a phase portrait to get full credit.**

$$y' = (2-y)^3(6-y)^2 = 0$$
$$(2-y)^3 = 0 \text{ or } (6-y)^2 = 0$$

Equilibrium → $y = 2$ $\quad$ $y = 6$
Solutions

$6 +$ Semi-stable → $y = 6$

$2 +$ → stable → $y = 2$

[b] If $y = f(x)$ is a solution of the DE such that $f(7) = 1$, what is $\lim_{x \to \infty} f(x)$?

$$\lim_{x \to \infty} f(x) = 2 \quad (7,1)$$

[c] If $y = g(x)$ is a solution of the DE such that $g(8) = 5$, what is $\lim_{x \to \infty} g(x)$?

$$\lim_{x \to \infty} g(x) = 2$$

Consider the DE $x^2 y'' - xy' + y = \sqrt{x}$.

[a] Is $y = 4\sqrt{x} + Ax + Bx \ln x$ a family of solutions of the DE ?

$4x^{1/2}$

$$y' = 2x^{-1/2} + A + B\left(\ln x + \frac{x}{x}\right)$$
$$y' = 2x^{-1/2} + A + B(\ln x + 1)$$
$$y'' = -x^{-3/2} + \frac{B}{x}$$

$$x^2 y'' - xy' + y = x^2\left(-x^{-3/2} + \frac{B}{x}\right) - x\left(2x^{-1/2} + A + B\ln x + B\right) + 4x^{1/2} + Ax + Bx\ln x$$
$$= -x^{1/2} + Bx - 2x^{1/2} - Ax - Bx\ln x - Bx + 4x^{1/2} + Ax + Bx\ln x$$
$$= x^{1/2} = \sqrt{x}$$

Yes, $y = 4\sqrt{x} + Ax + Bx\ln x$ is a family of solutions of the DE

[b] If the answer to [a] is "YES", solve the IVP consisting of the DE and the initial conditions $y(1) = 6$, $y'(1) = 2$. If the answer to [a] is "NO", write "SKIP" and skip this part.

$y(1) = 6$ $\qquad$ $y'(1) = 2$ $\qquad\qquad$ $y = 4\sqrt{x} + 2x - 2x\ln x$

$6 = 4 + A + 0$ $\qquad$ $2 = 2 + A + B$
$A = 2$ $\qquad\qquad$ $2 = 2 + 2 + B$
$\qquad\qquad\qquad$ $B = -2$

[MULTIPLE CHOICE] Write the letter of the correct answers in the spaces below.

ANSWERS:　**1** d ✓　**2** e ✓　**3** d ✓　**4** f ✓　**5** b ⃝　**6** b ⃝

1　How much of your learning does the instructor believe comes from your daily reading and homework combined ?

[a]　40%
[b]　50%
[c]　60%
[d]　70%
[e]　80%

2　Which statement below regarding tests (quizzes, midterms, final exam) is **false** ?

[a]　If you continue writing on your test after the stated ending time, you will receive a 0 for that test. ✓
[b]　There are no make-ups for missed quizzes. ✓
[c]　The instructor expects you to be able to identify and execute solutions on midterms more quickly than on quizzes because you should have had much more practice. ✓
[d]　If your tablet, phone, computer etc. makes an audible noise during a test, you will lose 10% of all points available on that test. ✓
[e]　If you cannot make the scheduled final exam time for any reason, your final exam can be rescheduled.

3　Proper use of the textbook for this class includes

[a]　understanding all the terminology used in the book ✓
[b]　working out the given examples yourself and checking that you are able to get the same results as the book ✓
[c]　reading the sections of the textbook before the corresponding lecture ✓
[d]　all of the previous answers [a], [b] and [c]
[e]　some, but not all, of the previous answers [a], [b] and [c]

4　If you score 120 points on Midterm 1,　140 points on Midterm 2　and　145 points on Midterm 3,
which midterm score(s) will be changed, and to what value ?
(HINT: You are encouraged to start studying regularly early in the quarter.)

[a]　Midterm 1's score will be changed to 145 (the highest midterm score) ✗
[b]　Midterm 1's score will be changed to $(120 + 140 + 145) \div 3 = 135$ (the average of all midterm scores) ✗
[c]　Midterm 1's score will be changed to $(120 + 140) \div 2 = 130$ (the average of Midterm 1's and Midterm 2's scores)
[d]　Midterm 1's score will be changed to $(120 + 145) \div 2 = 132.5$ (the average of Midterm 1's and the highest midterm's scores)
[e]　Midterm 1's score will be changed to $(120 + 145) \div 2 = 132.5$ and Midterm 2's score will be changed to $(140 + 145) \div 2 = 142.5$
(the average of each midterm's and the highest midterm's score)
[f]　no midterm scores will be changed

5　If you score 140 points on Midterm 1,　120 points on Midterm 2　and　145 points on Midterm 3,
which midterm score(s) will be changed, and to what value ?

[a]　Midterm 2's score will be changed to 145 (the highest midterm score)
[b]　Midterm 2's score will be changed to $(120 + 140 + 145) \div 3 = 135$ (the average of all midterm scores) ✓
[c]　Midterm 2's score will be changed to $(120 + 140) \div 2 = 130$ (the average of Midterm 2's and Midterm 1's scores)
[d]　Midterm 2's score will be changed to $(120 + 145) \div 2 = 132.5$ (the average of Midterm 2's and the highest midterm's scores)
[e]　Midterm 1's score will be changed to $(140 + 145) \div 2 = 142.5$ and Midterm 2's score will be changed to $(120 + 145) \div 2 = 132.5$
(the average of each midterm's and the highest midterm's score)
[f]　no midterm scores will be changed

6　Which statement below regarding attendance is **false** ?

[a]　Whenever you come into class (whether on time or late), you should sign in on the attendance spreadsheet right away. ✓
[b]　Arriving late on a quiz or midterm day will not be counted as late. ✗
[c]　Unexcused early departures are considered absences. ✓
[d]　If you have perfect attendance and classroom behavior for the first 7 weeks, and do not show up again after that, you will receive an F for the course.
[e]　Attendance policies will not apply to you if you score more than 80% on every midterm.